

# Klein Bottle

By Mariam Al-Hawaj

```
Speak["This program by Mariam Al-Hawaj draws pictures of a Klein Bottle"]
```

## ParametricPlot3D

```
Speak["Here, we draw the klein bottle using ParametricPlot3D which  
includes the function values for each of x, y, and z coordinates."]
```

```
a = 3;
```

```
ParametricPlot3D[{Cos[x] (a + Cos[x/2] Sin[y] - Sin[x/2] Sin[2 y]), Sin[x]  
(a + Cos[x/2] Sin[y] - Sin[x/2] Sin[2 y]), Sin[x/2] Sin[y] + Cos[x/2] Sin[2 y]},  
{x, -Pi, Pi}, {y, 0, 2 Pi}, Boxed -> False, Axes -> None, PlotPoints -> 100,  
MaxRecursion -> 5, Mesh -> None, NormalsFunction -> None] // Timing
```

```
{0.34375, }
```



# Initialization

```
Speak["To make it go faster, we made a list of values of  
x, y, and z and gave it the name klein. Then we used it in the  
ParametricPlot3D to draw the klein bottle. It definitely went faster."]
```

```
c = 3;  
klein = {Cos[u] (c + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]), Sin[u]  
  (c + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]), Sin[u/2] Sin[v] + Cos[u/2] Sin[2 v]};  
thePlot = ParametricPlot3D[klein, {u, -Pi, Pi}, {v, 0, 2 Pi},  
  Axes → None, Boxed → False, PlotPoints → 10, MaxRecursion → 5,  
  Mesh → None, NormalsFunction → None] // Timing
```

```
{0.171875, }
```

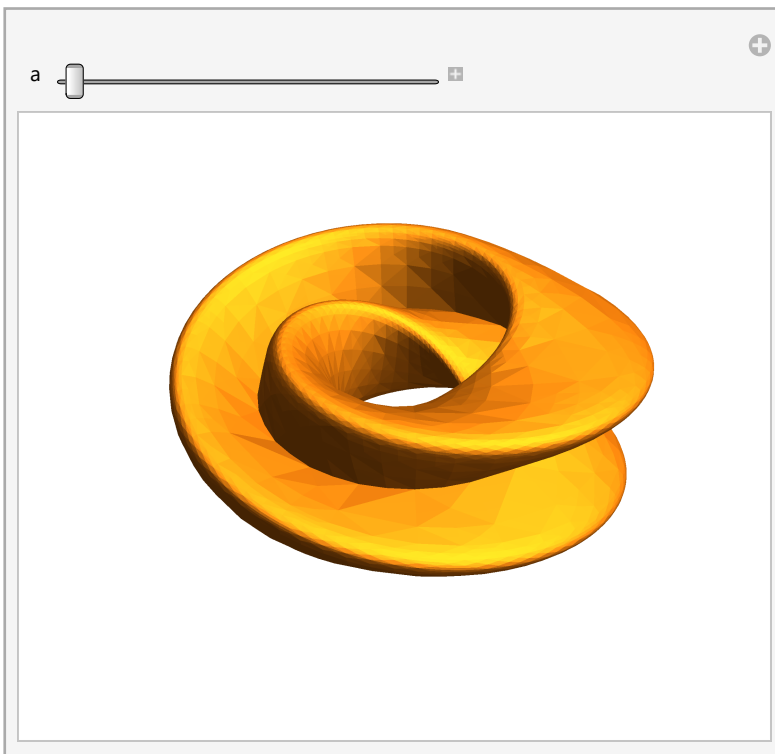


# Manipulation

```
Speak["Here, we used manipulate to show the klein bottle with different radii."]
```

```
a = 1;
```

```
Manipulate[  
  ParametricPlot3D[{Cos[u] (a + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]), Sin[u]  
    (a + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]), Sin[u/2] Sin[v] + Cos[u/2] Sin[2 v]},  
    {u, -Pi, Pi}, {v, 0, 2 Pi}, Axes → None, Boxed → False, PlotPoints → 10,  
    MaxRecursion → 5, Mesh → None, NormalsFunction → None], {a, -2, 2}]
```



# Animation

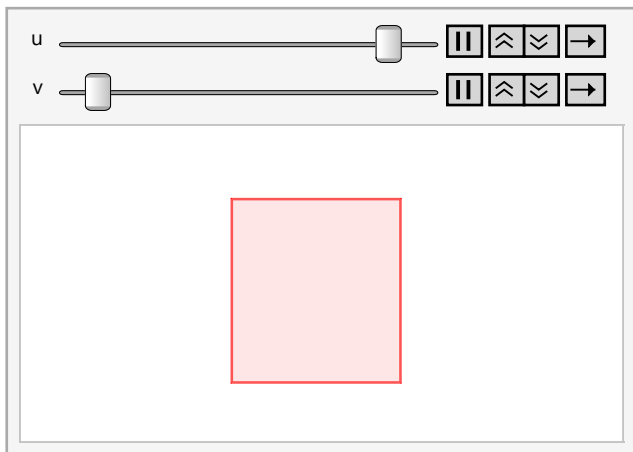
`Speak["We also tried to animate a small ball to slide on the klein bottle, and it didn't work as required. However, Mariam will continue to put sometime on this in the coming days to achieve the expected outcome."]`

```

b = 3;
ParametricPlot3D[{Cos[u] (b + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]),
  Sin[u] (b + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]),
  Sin[u/2] Sin[v] + Cos[u/2] Sin[2 v]}, {u, -Pi, Pi},
{v, 0, 2 Pi}, Axes → None, Boxed → False, PlotPoints → 10,
MaxRecursion → 5, Mesh → None, NormalsFunction → None]
Animate[ParametricPlot3D[{Cos[u] (b + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]), Sin[u]
  (b + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]), Sin[u/2] Sin[v] + Cos[u/2] Sin[2 v]},
{u, -Pi, Pi}, {v, 0, 2 Pi}, Axes → None, Boxed → False, PlotPoints → 10,
MaxRecursion → 5, Mesh → None, NormalsFunction → None]
Graphics3D[Point[{{Cos[u] (b + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]),
  Sin[u] (b + Cos[u/2] Sin[v] - Sin[u/2] Sin[2 v]),
  Sin[u/2] Sin[v] + Cos[u/2] Sin[2 v]}},
Boxed → False], {u, -2 Pi, 2 Pi}, {v, 0, 4 Pi}]

```





# Traditional Picture of the Klein Bottle

`Speak["Here, we used more complicated parameterization  
of the 3-dimensional immersion of the bottle to obtain the  
traditional picture. However, the picture doesn't show."]`

```
x[u_, v_] = -(2/15) Cos[u]
            (3 Cos[v] - 30 Sin[u] + 90 Cos^4[u] Sin[u] - 60 Cos^6[u] Sin[u] + 5 Cos[u] Cos[v] Sin[u]);
y[u_, v_] = -(1/15) Sin[u] (3 Cos[v] - 3 Cos^2[u] Cos[v] - 48 Cos^4[u] Cos[v] +
            48 Cos^6[u] Cos[v] - 60 Sin[u] + 5 Cos[u] Cos[v] Sin[u] -
            5 Cos^3[u] Cos[v] Sin[u] - 80 Cos^5[u] Cos[v] Sin[u] + 80 Cos^7[u] Cos[v] Sin[u]);
z[u_, v_] = (2/15) (3 + 5 Cos[u] Sin[u]) Sin[v];
ParametricPlot3D[{x[u, v], y[u, v], z[u, v]}, {u, 0, Pi}, {v, 0, 2 Pi}, Axes -> None,
                Boxed -> False, PlotPoints -> 10, MaxRecursion -> 5, Mesh -> None, NormalsFunction -> None]
```

# Another Try

```
Speak["Finally, we tried to include the functions within the  
ParametricPlot3D, but still not successful. I guess, more time  
needed for this to be completed by Mariam. Thank you for your time!"]
```

```
u = 0;
```

```
v = 0;
```

```
ParametricPlot3D[{- (2 / 15) * Cos[u]  
  (3 Cos[v] - 30 Sin[u] + 90 Cos4[u] Sin[u] - 60 Cos6[u] Sin[u] + 5 Cos[u] Cos[v] Sin[u]),  
 - (1 / 15) * Sin[u] (3 Cos[v] - 3 Cos2[u] Cos[v] - 48 Cos4[u] Cos[v] +  
  48 Cos6[u] Cos[v] - 60 Sin[u] + 5 Cos[u] Cos[v] Sin[u] -  
  5 Cos3[u] Cos[v] Sin[u] - 80 Cos5[u] Cos[v] Sin[u] + 80 Cos7[u] Cos[v] Sin[u]),  
 (2 / 15) * (3 + 5 Cos[u] Sin[u]) Sin[v]}, {u, 0, Pi}, {v, 0, 2 Pi},  
 Axes → None, Boxed → False, PlotPoints → 10, MaxRecursion → 5,  
 Mesh → None, NormalsFunction → None]
```